**Abstract**

It is now well known that in models with imperfect competition the choice of the normalization rule may have real effects. The scope and the reason of this result are investigated. I first show that a clear distinction must be made between imperfect competition based on quantity versus nominal strategic variables. In this latter case, the maximization of the shareholders' utility is not sufficient to cancel the real effects of the price normalization. In order for an equilibrium to be determined, a normalization rule describing the relation between strategic variables and aggregate demand (the so called Ford effects) is required.

In a second stage, I show that introducing money is not by itself sufficient to prevent the multiplicity of equilibria. Indeed, any non-monetary equilibrium can be replicated in a monetary economy by appropriately choosing the monetary rule. Hence, the introduction of money does not resolve the price normalization indetermination but transform it into an economic policy issue.

# Price Normalization and Monetary rule in imperfect competition general equilibrium models: A note.

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[1] I thank Torben Andersen and Henri Sneessens for their usefull comments. None of them is responsible for any errors or shortcomings. This text presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Ministers' Office, Science Policy Programming. The financial support of the 'Projet d'Action Concertée' 93/98-162 of the Ministery of Scientific Research of the Belgian French Speaking Community is gratefully acknowledged.

# 1 Introduction

The price normalization problem in Arrow-Debreu economies with imperfect competition was first raised by Gabszewicz and Vial [1972] in a quantity strategic variable model à la Cournot-Walras. In their paper, the authors realize that equilibrium real variables are affected by the particular price normalization used. Subsequent research by, e.g., Mas-Colell [1984] and Dierker and Grodal [1996] reveal that the problem is caused by the firms' limited rationality or computational ability: rational firms should care only about the welfare of their shareholders and this is not necessarily rendered by "nominal" profit maximization. The latter implies maximizing the value of the production expressed in terms of the numeraire. Instead firms should maximize their profit expressed in terms of the aggregate good consumed by their shareholders. If this is the case, the normalization problem disappears. Moreover, as Dierker and Grodal [1996] put it, "...there is no need for absolute prices in the theory of imperfect competition".

However Bohm [1994] shows in two examples that in an exchange economy *with price setting agents*, the normalization problem cannot be avoided solely by assuming fully rational firms. Section 2 develops this theme by distinguishing imperfect competition based on quantity strategic variables and on nominal strategic variables.

The starting point is a general equilibrium model with two rational strategic firms which maximize their real profits. Two versions of the model are proposed where the strategic variable used by each firm is either its output or its nominal price (i.e. respectively the Cournot-Nash and the Bertrand-Nash equilibria). It is shown that a normalization rule is not required to compute the Cournot-Nash equilibrium, while it is absolutely necessary when nominal strategic variables are used. Without a normalization rule, which always necessarily defines a relation between strategic nominal variables and nominal aggregate demand (the so called "Ford effect" after d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1990] and d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1996]), the model cannot be solved.

As often suggested, the problem of the multiplicity of equilibria must be solved by use of a "natural" numéraire. In this line, a straightforward numéraire is money, as suggested by Böhm. Section 3 enlarges the preceding set up to include real money in the consumers' utility function. However, the introduction of money does not resolve the normalization problem. Indeed, we show that to any particular normalization rule in a non-monetary model corresponds a monetary rule which replicates the real outcome of the previous model in a monetary economy. In particular, assuming money supply to be exogenous is restrictive and corresponds to a particular price normalization rule in a non-monetary model where nominal aggregate demand is normalized to a constant. Many other monetary rules (determining how money supply reacts to nominal strategic variables) are possible. However, monetary models allow to endogenize the determination of the normalization rule which can be set by the monetary authorities. Hence, the money can provide a non-arbitrary way out of the ad hoc choice of a normalization rule.

# 2 The model

The objective of this section is to compare the normalization problem in a general equilibrium model with either quantity setting agents or price setting agents. The present model serves only as an example to illustrate my argument. After the description of consumers and firms, I compare the Cournot-Nash equilibrium to the Bertrand-Nash equilibrium.

## 2.1 Consumers

All consumers (indexed by $j$) are identical and have Cobb-Douglas preferences over two goods $C_i$, $i = 1, 2$ and supply one unit of labor if the real wage is larger than their disutility of work $v$ (in which case $h_j = 1$):

$$U_j = \prod_{i=1}^{2} C_i^{1/2} - v h_j, \ \forall j \tag{1}$$

They maximize their utility subject to their budget constraint. As preferences are homothetic, a redistribution of total income has no effect on aggregate demand. Hence, I consider only the aggregate budget constraint $\Omega$ defined as

$$\Omega = \sum_i P_i C_i = \sum_i P_i Y_i \tag{2}$$

where the demand for goods is:

$$C_i = \frac{1}{2} \frac{\Omega}{P_i}, \ i = 1, 2. \tag{3}$$

The true consumption price index is defined as

$$Q = \prod_{i=1}^{2} P_i^{1/2}. \tag{4}$$

## 2.2 Firms

Each good is produced by a monopolistic firm using labor as the only factor in a Cobb-Douglas production function:

$$Y_i = L_i^{\alpha} \ , 0 < \alpha < 1, \ i = 1, 2. \tag{5}$$

The labor market is competitive and labor supply is infinitely elastic at the *real* wage[1] $v$

$$\frac{W}{Q} = v. \tag{6}$$

[1] Note that there is no nominal rigidity nor money illusion of any kind in the present setting.

For simplicity I assume that the two firms have disjoint groups of shareholders (see also Hart [1985] or Dierker and Grodal [1996]) and that they set their price $P_i$ so as to maximize the utility of their shareholders. As the price of the consumption bundle of the shareholders is given by $Q$, each firm maximizes its *real* profits, that is, the value of its profit expressed in terms of the aggregated good:

$$\frac{\Pi_i}{Q} = \frac{P_i}{Q} C_i - vL_i, \ i = 1, 2 \tag{7}$$

with $L_i = C_i^{1/\alpha}$. With this formulation, the possible real effects of a price normalization cannot be due to "irrational" behavior of the firms.

Note that firms maximize their profit at a given *real* wage. Assuming instead that firms reason at given *nominal* wages would strengthen our results. I will come back to this case later.

Let us now compare quantity and price setting equilibria. I want to show that price normalization is not required to compute the Cournot-Nash equilibrium (in fact, the Dierker and Grodal [1996] result) while it is absolutely necessary to do so with nominal strategic variables. Moreover, the price normalization affects the real outcome only in this latter case.

### 2.3 Cournot-Nash equilibrium

I start with the quantity setting equilibrium where each firm sets its production so as to maximize its real profit given the output of the other firm. If each firm $i$ maximizes its real profit with respect to $Y_i$, taking $Y_k$ as given ($Y_k = \overline{Y}_k$), its objective simply takes the form:

$$\frac{\Pi_i}{Q} = \frac{P_i}{Q} Y_i - v \left(Y_i\right)^{1/\alpha}, \ i = 1, 2. \tag{8}$$

Due to the Cobb-Douglas preferences, production is the same for each firm: $P_i Y_i = P_j \overline{Y}_j = \frac{\Omega}{2}$ or $\frac{P_i}{P_j} = \frac{\overline{Y}_j}{Y_i}$. Therefore (8) can be written as

$$\frac{\Pi_i}{Q} = \left(Y_i \overline{Y}_k\right)^{1/2} - v \left(Y_i\right)^{1/\alpha}, \ i = 1, 2 \tag{9}$$

which does not depend on any nominal value. Therefore price normalization is not required to obtain the Nash equilibrium. However, this will no longer be the case with nominal strategic variables.

The interpretation of (9) is straightforward: firm $i$ knows that any increase in its production $Y_i$ will also raise real aggregate revenue $\frac{\Omega}{Q} = 2\left(Y_i \overline{Y}_k\right)^{1/2}$ and thus increase demand for its own product by half. This effect is usually referred to as the "Ford Effect" after d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1990] and d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1996] since a firm acknowledges that an increase in its production will raise the aggregate demand and thus the demand for its own demand. The demand for good $i$ depends on production $Y_{k \neq i}$ (i.e. the elasticity of real aggregate demand $\frac{\Omega}{Q}$ to the strategic variable $Y_i$ is smaller than

1): this fact is the root of the sub-optimality of the Cournot-Nash equilibrium as each firm does not internalize the positive effect on the other firm's profit when it raises it own production.

The first order condition for profit maximization leads to the "best response" quantity setting rule

$$Y_i^* = \left(\frac{2v}{\alpha}\right)^{\frac{-2\alpha}{2-\alpha}} Y_k^{\frac{\alpha}{2-\alpha}} \tag{10}$$

And the Nash equilibrium of the game à la Cournot is:

$$Y_i^* = \left(\frac{2v}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}}, \ i = 1, 2 \tag{11}$$

and

$$P_i^* = \frac{\Omega}{2}\left(\frac{2v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}, \ i = 1, 2. \tag{12}$$

This clearly shows that using quantity as the strategic variable implies that the price normalization rule plays does not affect the solution, as the relative price is not affected by it.

Compared to the competitive solution,

$$P_i^c = \frac{\Omega}{2}\left(\frac{v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}, \tag{13}$$

$$Y_i^c = \left(\frac{v}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}}, \ i = 1, 2, \tag{14}$$

the Cournot-Nash equilibrium corresponds to higher absolute prices and lower production for both firms.

## 2.4 Bertrand-Nash equilibrium

Let us write the profit equation (7) using (3), the demand for good $i$:

$$\frac{\Pi_i}{Q} = \frac{1}{2}\frac{\Omega}{Q} - v\left(\frac{1}{2}\frac{\Omega}{P_i}\right)^{1/\alpha}, \ j = 1, 2. \tag{15}$$

Each firm $j$ maximizes $\frac{\Pi_i}{Q}$ with respect to $P_i$ taking $P_{k\neq i}$ as given but takes all other effects into account. In particular, each firm takes into account the possible repercussion of its decision on the price index $Q$ and on the nominal aggregate demand $\Omega$ (i.e. the Ford effects).

The first order conditions for profit maximization yield the following best response price setting rule for each firm:

$$P_i^* = \left[\gamma_i \frac{v}{\alpha} Q\right]^{\alpha} \left[\frac{\Omega}{2}\right]^{1-\alpha} \tag{16}$$

$$= \left[\gamma_i \frac{v}{\alpha}\right]^{\frac{2\alpha}{2-\alpha}} P_{k\neq i}^{\frac{\alpha}{2-\alpha}} \left[\frac{\Omega}{2}\right]^{1-\frac{\alpha}{2-\alpha}}, \quad i = 1, 2 \tag{17}$$

where $\gamma_i$ is the mark-up over marginal cost[2] for firm $i$ defined as

$$\gamma_i = \frac{1 - \epsilon[\Omega, P_i]}{\epsilon[Q, P_i] - \epsilon[\Omega, P_i]} \tag{18}$$

where $\epsilon[X, Y]$ denotes the elasticity of $X$ with respect to $Y$.

By solving (16) one can obtain the equilibrium prices:

$$P_i^* = \frac{\Omega}{2} \left( \gamma_i^{1-\frac{\alpha}{2}} \gamma_k^{\frac{\alpha}{2}} \frac{v}{\alpha} \right)^{\frac{\alpha}{1-\alpha}}, \; i \neq k = 1, 2. \tag{19}$$

Equilibrium consumptions are given by:

$$C_i^* = \left( \gamma_i^{1-\frac{\alpha}{2}} \gamma_k^{\frac{\alpha}{2}} \frac{v}{\alpha} \right)^{\frac{-\alpha}{1-\alpha}}, \; i \neq k = 1, 2. \tag{20}$$

This solution depends on the value of the elasticities in $\gamma_i$, namely $\epsilon[Q, P_i]$ and $\epsilon[\Omega, P_i]$. Obviously, the price index elasticity $\epsilon[Q, P_i]$ is equal to $1/2$ by (4). However, determination of the elasticity of aggregate demand is more problematic as it is not defined unless we add a normalization rule to the model.

There are 5 nominal variables: the aggregate expenditure $\Omega$ and 4 prices (i.e. $P_1, P_2, Q, W$). On the other hand, there are only 4 economic relations to determine these 5 variables, namely, the two price setting equations (19), the price index definition (4) and the labor market competitive wage equation (6). Hence, there is one relation too few in order to determine the *level* of nominal variables and in particular the level of the aggregate demand which affects all other nominal variables. However, note that a price normalization is required not only to define the general price *level*, but also, and more crucially, to define the Ford effects $\epsilon[\Omega, P_i]$ and the $\gamma_i$ parameters. This will be made clearer in the next paragraph.

In the most general way, a normalization rule is a relation linking (some of) the nominal variables of the model to a constant. As non-strategic nominal variables (such as competitive prices) can always be expressed in a reduced form as a function of strategic variables and aggregate demand $\Omega$, *in fine* only these latter variables must enter the normalization rule. That means that behind the choice of a given "natural" numéraire such as a particular competitive price, one determines a more complex normalization relation between strategic decision variables and aggregate demand.

In order to clarify the exposition, I propose the following particular normalization rule

$$P_1^{\delta_1} P_2^{\delta_2} \Omega^{\delta_3} = G, \; \delta_i \geq 0, \; i = 1, 2, 3 \tag{21}$$

where $G$ is any (strictly) positive constant, wich allows for several interesting polar cases, such as $\Omega = 1$ (if $\delta_1 = \delta_2 = 0$ and $G = 1$); $P_1 = 1$ (if $\delta_1 = 1$ and $\delta_j = 0 \; \forall j \neq 1$ and $G = 1$); $Q = 1$

[2] The marginal cost $MC_i$ is equal to $\frac{\partial(WL_i)}{\partial Y_i} = \frac{W}{\alpha} Y_i^{\frac{1-\alpha}{\alpha}} = \frac{v}{\alpha} Q \left( \frac{1}{2} \frac{\Omega}{P_i} \right)^{\frac{1-\alpha}{\alpha}}$. Defining $P_i = \gamma_i MC_i$ leads to (16).

(if $\delta_1 = \delta_2 = 1/2$ and $\delta_3 = 0$, $G = 1$) and $W = 1$ (if $\delta_1 = \delta_2 = 1/2$ and $\delta_3 = 0$, $G = 1/v$). Note that the last two cases are equivalent since imposing $W = 1$ is equivalent to imposing $Q = 1/v$ (due to the perfectly elastic labor supply in the present model).

Several constraints bear on the $\delta_i$ parameters for (21) to be a proper normalization rule. First, it must not be homogeneous of degree 0 in all its variables (i.e. $\sum \delta_i \neq 0$), otherwise it would fix some *relative* price and not the general price level. As a result, the sum of the exponents of (21) is usually normalized to 1: $\sum \delta_i = 1$.

Second, $\delta_3$ must be different from 0 as a normalization rule where $\Omega$ is absent (i.e. $\delta_3 = 0$) is not acceptable as it comes down to imposing a direct constraint on the strategic price variables $P_1$ and $P_2$. This is in contradiction with the price setting behavior of the strategic agents and with the concept of Nash equilibrium in nominal variables (as each firm must reason at a *given* price for the other firm). Hence we must drop those cases by imposing $\delta_3 \neq 0$. This means that aggregate demand $\Omega$ will *always* necessarily be part of the normalization rule (either alone or with strategic variables)[3].

The solution to the model depends on the choice of the $\delta_i$ parameters as the elasticity of the aggregate demand to the sectorial price $P_j$ is given by:

$$\epsilon[\Omega, P_i] = -\frac{\delta_i}{\delta_3}, \quad i = 1, 2. \tag{22}$$

Hence, the price mark-up in sector $j$ depends on the parameters $\delta_3$ and $\delta_i$ of the normalization rule through (18):

$$\gamma_i = \frac{\delta_3 + \delta_i}{\frac{\delta_3}{2} + \delta_i}, \; i = 1, 2. \tag{23}$$

Therefore the equilibrium can be determined only conditionally to a particular normalization rule and a wide range of equilibria are possible. Note that with non-negative $\delta_i$, the markups can range from 2 (when $\delta_i = 0, \; i = 1, 2$) to the competitive outcome where $\gamma_i = 1$ when $\delta_i \to \infty, \; i = 1, 2$.

Suppose for instance that the following (common) normalization rule $\Omega = 1$ is assumed. Then the elasticity $\epsilon[\Omega, P_i] = 0$ and we have for the markups $\gamma_i = 2$ ($\forall j$), as in the Cournot-Nash equilibrium. This is due to the fact that when $\Omega \equiv 1$ the Ford effects are similar in both types of equilibrium. When strategic agents use a real strategic variable, the Ford effects are defined independently of the price normalization and of all nominal variables. If one of the sectors (producing half of the whole economy's production) raises its (real) production level, the real aggregate demand is increased by half, independently of any possible price normalization.

[3] In the present model, it is not possible to normalize the competitive wage $W = 1$ as this results in imposing a direct relation between both of the strategic prices $P_1$ and $P_2$. For this to be possible it would be necessary that the equilibrium competitive wage also depends on aggregate demand $\Omega$, i.e. that labour supply is not perfectly elastic.

On the contrary, with nominal strategic variables, an increase in the (real) production of a sector (i.e. a reduction in its sectorial price) may be linked in various ways with the real aggregate demand $\frac{\Omega}{Q}$. It is only when $\Omega \equiv 1$ in the price setting game that the elasticity of the real aggregate demand $\frac{\Omega}{Q}$ to sectorial quantity is equal in both strategic games[4].

The following proposition states a sufficient condition for price normalization to matter, even though firms are rational.

**Proposition 1** *A sufficient condition for a price normalization to have real effects in a general equilibrium model is that some strategic agent(s) use(s) (a) nominal decision variable(s).*

Note that if both goods markets were competitive[5], both mark-ups $\gamma_1$and $\gamma_2$ would be equal to 1 and price normalization would not affect the real outcome of the model. Hence, at least one non-competitive agent is required for normalization to be effective to alter the real outcome. Moreover, if only one firm is non-competitive (and all other markets are competitive), normalization matters provided that the non-competitive firm sets its price for a given “absolute” price level in at least one of the competitive markets. Indeed, if an agent reasons at given relative prices for *all* the other (competitive and strategic) agents, then his own relative price is also necessarily fixed and his decision bears on a real variable. Therefore, price normalization cannot affect his decision making. This is illustrated in the appendix. Hence, the strategic agent must reason with at least one absolute price level given in order to be affected by the price normalization. This is necessarily the case when there are several strategic agents using a nominal decision variable.

We have seen that a normalization rule is the necessary definition of Ford effects which affect the relations between the model's nominal variables. This is so whether strategic agents do take the Ford effects into account *or not*. Simply, if they do not, their behaviour will not be optimal (i.e. rational), and the computed Nash equilibrium will only be an approximation of the true solution of the model. Now, if one considers myopic strategic agents neglecting Ford effects, the approximation error will depend on the normalization rule used in the model. The larger the existing Ford effects, the larger the approximation errors if strategic agents ignore them when making their decisions. For instance, if the nil Ford effect normalization rule is chosen (i.e. $\epsilon[\Omega, P_i] = 0 \; \forall i$), there is no irrationality to ignore Ford effects as there is none, and the exact

[4]The elasticity of real aggregate demand $\frac{\Omega}{Q}$ to price is equal to $(-1/2)$ when $\Omega \equiv 1$ in the price setting game and to $1/2$ in the quantity setting game. As the own demand price elasticity is equal to $-1$ here, quantity and price are inversely related and the Ford effects are similar when $\Omega \equiv 1$.

[5]In this case, the price $P_i$ that clears the market for good $i$ is equal to

$$P_i = \left[\frac{v}{\alpha}Q\right]^{\alpha}\left[\frac{\Omega}{2}\right]^{1-\alpha}, \quad i = 1,2 \tag{24}$$

which corresponds to equation (16) when $\gamma_i = 1$.

solution is obtained even by "myopic" agents. For this reason the normalization rule setting nominal aggregate demand to a constant ($\Omega \equiv 1$) should be preferred when Ford effects has to be neglected in order to derive the solution of the model.

# 3 Money as a price normalization device

It is often claimed, as in Bohm [1994], that money suppresses the normalization problem by providing a numéraire. As such, money is very useful. However, this is only part of the story, as I will show in the present section that the way money is supplied affects the real equilibrium in about the same way as a price normalization rule. Any arbitrary price normalization rule (in a non-monetary model) corresponds to a money supply rule in the corresponding "extended model" (where real money holdings appear in the consumers' utility) which replicates the real outcome of the non-monetary economy. Böhm is right in the sense that introducing money provides an *indirect* way to cope with the arbitrary normalization problem as it allows to place the choice of the normalization rule in a monetary policy perspective.

The interest of this result lies in the fact that the equivalence between a non-monetary model and its monetary counterpart is generally not acknowledged. The paper of d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1996] provides a good illustration of this. The authors compare the approximation errors made by ignoring the Ford effects in the Dixit-Stiglitz model and in its extended monetary version. Because the price normalization in the non-monetary model does not correspond to the monetary rule used in the monetary model, their results differ in each case. However, the mere presence of money in the extended model is not responsible for this conclusion. It is instead because there is no correspondence between the price normalization rule (in the non monetary model) and the monetary rule (in its monetary extension) that results differ accross the two models. I will come back to this after having made the point with the simple model of the preceding section.

To introduce money, I assume that consumer $i$ derives some utility from holding real balances $\frac{M_i}{Q}$. The utility derived from money can be seen as a proxy for future consumption or as a simple way to take transaction costs into account (see Feenstra [1986]). Moreover, I make the (very common) assumption of a Cobb-Douglas utility structure for aggregate consumption and money holdings:

$$U_j = \left[C_1^{1/2} C_2^{1/2}\right]^c \left(\frac{M_j}{Q}\right)^{1-c} - vh_j, \ \forall j \tag{25}$$

where $Q$ is still the same consumption price index already defined in (4).

The aggregate nominal budget constraint $\Omega$ is now defined as

$$\Omega = M + \sum_i P_i Y_i. \tag{26}$$

Thus the demand for goods is given by:

$$C_i = \frac{c}{2}\frac{\Omega}{P_i},\ i = 1, 2 \tag{27}$$

and money demand is equal to $M^d = (1 - c)\Omega$.

At equilibrium $Y_i = C_i\ \forall i$ and by Walras law money demand equal money supply $M^d = M$. Hence, one can introduce (27) in (26) to write the budget constraint as

$$\Omega = \frac{1}{1 - c}M. \tag{28}$$

$\Omega$ is simply proportional to $M$ and the price normalization is not required anymore to define the non-cooperative outcome. Instead, money supply $M$ replaces the price normalization as a way to set the elasticity between $\Omega$ and both prices. A *monetary rule* defines the way $M$ reacts to the strategic variables $P_i$ and will affect the outcome of the non-cooperative price setting game.

The following (admittedly ad hoc) monetary rule is proposed as an example in the rest of the analysis:

$$M = D^{1+\mu_1+\mu_2} P_1^{-\mu_1} P_2^{-\mu_2} \tag{29}$$

with all $\mu_i$ non negative. Money supply is determined by an exogenous (discretionary) component[6] $D$ and the level of both goods prices.

As in the previous model, the two firms have disjoint groups of shareholders (see also Hart [1985] or Dierker and Grodal [1996]). They set their price $P_i$ so as to maximize the sum of their *real* profits plus the real balance of their shareholders[7] $\frac{M_i}{Q} = \sum_{j \in i} \frac{M_j}{Q}$:

$$\frac{\Pi_i}{Q} = \frac{P_i}{Q}C_i - vL_i + \frac{M_i}{Q},\ i = 1, 2. \tag{30}$$

Let $\theta_i$ denote the share of the total money stock held by the shareholders of firm $i$ ($\theta_i = \frac{M_i}{M}$). Then, using the demand equation (27), the objective of the firm can be written as:

$$\frac{\Pi_i}{Q} = \left(\theta_i + \frac{1}{2}\frac{c}{1-c}\right)\frac{M}{Q} - v\left(\frac{1}{2}\frac{c}{1-c}\frac{M}{P_i}\right)^{1/\alpha},\ i = 1, 2. \tag{31}$$

The first order conditions lead to the following equilibrium prices and quantities:

$$P_i^* = \frac{1}{2}\frac{cM}{1-c}\left[\left(\frac{\varphi_i}{\eta_i}\right)^{1-\alpha/2}\left(\frac{\varphi_k}{\eta_k}\right)^{\alpha/2}\frac{v}{\alpha}\right]^{\frac{\alpha}{1-\alpha}},\ i = 1, 2 \tag{32}$$

$$C_i^* = \left[\left(\frac{\varphi_i}{\eta_i}\right)^{1-\alpha/2}\left(\frac{\varphi_k}{\eta_k}\right)^{\alpha/2}\frac{v}{\alpha}\right]^{\frac{-\alpha}{1-\alpha}},\ i = 1, 2 \tag{33}$$

[6] The exponent of the exogeneous factor $D$ is chosen so as to ensure homogeneity of degree 1 in all of the nominal variables at equilibrium.

[7] We use the real money balance in order to reassure the reader that no "irrationality" is introduced in the firm's behavior in order to obtain our results. Accordingly, the firm maximizes the (indirect) utility of its shareholders.

with

$$1 < \varphi_i = \frac{1+\mu_i}{1/2+\mu_i} \leq 2 \tag{34}$$

and

$$1 \leq \eta_i = 1 + \left(\frac{1-c}{c}\right)\theta_i \leq \frac{1}{c}.$$

Note that all nominal variables are homogenous of degree 1 in the money stock. Hence, money is neutral in the classical sense. But the monetary rule affects nominal and relative prices through the $\varphi_i$ parameters. Thus, while the money *stock* is neutral, the monetary *rule* is not!

If both $\theta_i = 0$ (i.e. if both firms do not take the effect of the price setting on the real balances of their shareholders into account), the solution is equivalent to the one obtained in the non-monetary model as the equilibrium becomes

$$C_i^* = \left(\varphi_i^{1-\frac{\alpha}{2}} \varphi_k^{\frac{\alpha}{2}} \frac{v}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}}, \ i \neq k \in \{1,2\}. \tag{35}$$

It is then possible to replicate the non-monetary equilibrium corresponding to a particular normalization rule $(\delta_1, \delta_2, \delta_3)$ by a monetary equilibrium where all firms (and more generally, all strategic agents) do not take the consequences of their decisions on the real money holdings of their shareholders into account and where the associated monetary rule is given by $(\mu_1, \mu_2) = \left(\frac{\delta_1}{\delta_3}, \frac{\delta_2}{\delta_3}\right)$.

In case firms are not "irrational" and take the real money holdings of their shareholders into account (i.e. $\theta_1 + \theta_2 = 1$), firms set lower equilibrium prices in order to limit the erosion of real money holdings. This implies higher production and consumption levels. In this case, the monetary rule which replicates a particular price normalization in the non-monetary economy is given by:

$$\frac{\varphi_i}{\eta_i} = \gamma_i, \ i = 1, 2. \tag{36}$$

We can now apply the preceding result to the paper of d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1996]. The authors present two different normalizations of the Dixit-Stiglitz model: one in the simple non-monetary version, one in its extended monetary version. In the non-monetary version, they choose to normalize the price of the non-produced good (e.g. labor) to 1 while in the monetary extended version they set the money stock equal to a constant. If instead, they had chosen to normalize the aggregate income or the wage to a constant in both versions, their results would have been similar for each version.

# 4 Concluding remarks

The two models described are by far too simplistic to derive general conclusions. However, because of their simplicity they provide a clear insight on the possible generality of the results.

The maximization of the shareholders' utility is not sufficient to always cancel the real effects of the price normalization. Indeed, a sufficient condition for price normalization to affect the real equilibrium is that strategic agents use nominal decision variables. This non-monetary equilibrium can be replicated in a monetary economy by appropriately choosing the monetary rule (defining how the money supply reacts to strategic nominal variables). Hence, the introduction of money does not resolve the price normalization indetermination but allows to endogenize it, as the choice of the monetary rule is an economic policy issue. This has an important implication for monetary policy: though money is still neutral in the classical sense (i.e. a once and for all variation in the money supply has no real effect), a change in the monetary rule affects the real equilibrium outcome.

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# A Appendix: Real price setting

We show that when a firm sets its price at given *relative* prices for all other markets, the normalization problem disappears. Suppose for instance that firm $i$ reasons with $\frac{P_i}{Q}$ given (and fixed e.g. at $k$). In this case, all relative prices are given to the firm, including its own relative price $\frac{P_i}{P_k} = \frac{1}{k^2}$. Hence, the firms do not start a counter effective price overbidding. However, by setting $P_i$, firm $i$ can determine the level of aggregate demand expressed in its own product $\frac{\Omega}{P_i}$. Simply, each firm does not try to improve its relative price but instead relies only on the "Ford" effect, in the sense that it tries to raise the purchasing power of the consumers expressed in its own good by changing the general price level. In this case, the firm's objective (15) becomes

$$\max_{P_i} \frac{\Pi_i}{Q} = \frac{1}{2k}\frac{\Omega}{P_i} - v\left(\frac{1}{2}\frac{\Omega}{P_i}\right)^{1/\alpha} \tag{37}$$

$$= \frac{1}{k}Y_i - vY_i^{1/\alpha},\ i = 1, 2. \tag{38}$$

As in the Cournot-Nash profit maximization (9), there is no nominal variable in the profit function and a price normalization is not required to define the equilibrium. However, the present equilibrium is different from the Cournot-Nash equilibrium. As each firm reasons here with its relative price given, a change in its own price implies a proportional change in the other price, so that the production levels change proportionately in both sectors. Hence, the consequence of a price rise $P_i$ on firm $i$ is identical to the consequence on firm $j$. That is, there is full internalization by each firm which implicitly takes the negative spillover it casts on the other firm when it raises its own price fully into account. As a result, the non-cooperative equilibrium is similar to the competitive equilibrium.